Effects of viscosity, temperature, and rate of rotation on pressure generated by a controlled-clearance piston gauge

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The calculation of the pressure generated by a controlled-clearance piston gauge depends upon the jacket pressure corresponding to zero clearance between the piston and cylinder P_z. The dependence of P_z on the viscosity of the pressure transmitting fluid, the temperature, and the rate of piston rotation have been measured. The four fluids used in this study have viscosities ranging from 0.7 to more than 10 000 cP. The value of P_z is nearly independent of viscosity below 60 cP. Above 60 cP, P_z depends strongly upon viscosity. Variations of P_z with temperature and rate of rotation are more severe at higher viscosities. The results suggest that the best choice of fluid is the one having the lowest viscosity at the operating pressure. Such a fluid can be selected on the basis of having the most nearly linear plot of P_z as a function of pressure of the candidate fluids. These results are also a clear indication that, for the most accurate pressure measurements, a controlled-clearance piston gauge must be characterized using the same operational and environmental conditions with the same fluid as are used in normal operation.

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INTRODUCTION

Reliability and accuracy of pressure measurements[1–5] are of prime importance in the field of chemical engineering, the power generation industry, oil exploration, aircraft operation, and materials processing. The most commonly used mechanical devices for accurate determination of pressure above 1 atm are piston gauges.[4–7] Essentially a piston gauge is a piston of known area fitted into a matching cylinder filled with fluid, loaded with known weights, and rotated with respect to the cylinder to relieve friction. The total upward force due to the pressure in the system acting on the area of the piston is balanced against the downward gravitational force of the weights. The pressure is defined as the ratio of the total downward force to the area. As the force can be measured with greater accuracy than the area, the limitation of the accuracy of the gauge is in the determination of the area and how it changes with pressure due to elastic distortion of the piston and cylinder.

One method of reducing the effect of the distortion is to provide a means of applying an auxiliary pressure to the outer surface of the cylinder to control the width of the annulus between the piston and the cylinder, as in the controlled-clearance piston gauge.[4–8] As the effect on the area due to the elastic distortion of the cylinder is several times that due to the piston, controlling the distortion of the cylinder can provide a significant reduction of inaccuracy. Furthermore, the ability to control the clearance allows one to obtain the best operating conditions for the particular pressure transmitting fluid used.

In a properly operating piston gauge, there is always fluid leaking past the piston which provides lubrication and results in the piston slowly falling into the cylinder. Measurement of the fall rate as a function of load and clearance-controlling pressure (jacket pressure P_j) is a necessary process in the characterization of a controlled-clearance piston gauge in order to determine the appropriate values of P_j to be used in operation. In the past, the fall rate was assumed to be independent of the pressure transmitting fluid, provided the fluid did not freeze or provided that the lubricating film between the piston and the cylinder was maintained.[6,7] Recently, Newhall, Ogawa, and Zilberstein[8] have shown the fall rates to vary considerably with different fluids.

Because of the importance of the controlled-clearance piston gauge as a primary pressure standard, we have made a systematic study of the effect of fluid viscosity, temperature, and speed of rotation on the fall rates.

I. THEORY

The pressure generated by a controlled-clearance piston gauge operating at its reference level is given by[4]

$$P = \frac{Mg(1 - \rho_a/\rho_m) + \gamma C}{A_0(1 + bP)[1 + (\alpha_c + \alpha_p)(T - T_r)][1 + d(P_z - P_j)]}, \quad (1)$$

where

M is the mass of the piston and the total load applied to the piston.

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TABLE I. Total uncertainty of pressure using H0,100 at 177 rpm and 297 K.

Parameter x_i	Differential $\frac{1}{P}\frac{\delta P}{\delta x_i}$	Differential value	Parameter uncertainty (dx_i)	Fractional uncertainty in terms of pressure $\frac{1}{P}\frac{\delta P}{\delta x_i}dx_i$
$M(2.805\,79\times10^{2}$ kg$)$	$1/M$	3.564×10^{-3}/kg	5.6×10^{-4} kg	2.0×10^{-6}
$g(9.801\,01$ m/s$^2)$	$1/g$	1.020×10^{-1} s^2/m	2×10^{-5} m/s^2	2.0×10^{-6}
$\rho_a(1.18$ kg/m$^3)$	$1/\rho_m$	1.190×10^{-4} m^3/kg	5×10^{-2} kg/m^3	5.6×10^{-6}
$\rho_m(8.4\times10^{3}$ kg/m$^3)$	ρ_a/ρ_m^2	1.672×10^{-8} m^3/kg	1×10^{2} kg/m^3	1.7×10^{-6}
$\gamma(3.1\times10^{-2}$ N/m$)$	C/gM	3.275×10^{-6} m/N	3×10^{-3} N/m	0.0×10^{-6}
$C(9.007\,61\times10^{-3}$ m$)$	γ/gM	1.127×10^{-5}/m^2	3×10^{-6} m	0.0×10^{-6}
$A_0(6.454\,091\,4\times10^{-6}$ m$^2)$	$1/A_0$	1.549×10^{5}/m^2	3.4×10^{-10} m^2	52.7×10^{-6}
$\alpha_c(1.2\times10^{-5}$/K$)$	$(T-Tr)$	0.4 K	6×10^{-7}/K	0.2×10^{-6}
$\alpha_p(4.3\times10^{-6}$/K$)$	$(T-Tr)$	0.4 K	2×10^{-7}/K	0.1×10^{-6}
$(T-Tr)(Tr=297$ K$)$	$(\alpha_c+\alpha_p)$	1.63×10^{-5}/K	0.05 K	0.8×10^{-6}
$b(-5.08\times10^{-13}$/Pa$)$	P	4×10^{8} Pa	5×10^{-15}/Pa	2.0×10^{-6}
$P(4\times10^{8}$ Pa$)$	b	5.08×10^{-13}/Pa	4×10^{5} Pa	0.2×10^{-6}
$P_z(3.41\times10^{8}$ Pa$)$	d	7.00×10^{-12}/Pa	6.0×10^{6} Pa	42.0×10^{-6}
$d(7.00\times10^{-12}$/Pa$)$	(P_z-P_j)	1.195×10^{8} Pa	2.5×10^{-13}/Pa	29.9×10^{-6}
$P_j(2.21\times10^{8}$ Pa$)$	d	7.00×10^{-12}/Pa	2.6×10^{5} Pa	1.8×10^{-6}

Root-mean-square of all terms of the total differential

$$\frac{dP}{P}=\left[\sum_1^i\left(\frac{1}{P}\frac{\delta P}{\delta X_i}dX_i\right)^2\right]^{1/2}\simeq 74 \text{ ppm}$$

g is the local acceleration due to gravity.
ρ_a is the density of the air.
ρ_m is the density of the weights.
γ is the surface tension of the fluid.
C is the circumference of the piston where it emerges from the fluid.
A_0 is the area of the piston at the reference temperature T_r and at atmospheric pressure.
α_c is the linear thermal expansion coefficient for the cylinder.
α_p is the linear thermal expansion coefficient for the piston.
T is the temperature of the piston and cylinder at the time of the pressure measurement.
T_r is the reference temperature at which A_0 was determined.
b is the pressure coefficient of the piston.
d is the pressure coefficient of the cylinder.
P_j is the jacket pressure during operation.
P_z is the jacket pressure at which the clearance between the piston and cylinder goes to zero.

The value of b is calculated from elastic theory using

$$b=(3\mu-1)/E, \tag{2}$$

where μ is Poisson's ratio and E is the modulus of elasticity for the piston.

Values for d can be obtained from calculations based on elastic theory or from direct measurements on the piston gauge. The expression for the calculation is[4]

$$d=2w^2/E(w^2-1), \tag{3}$$

where w is the ratio of outer to inner radius of the cylinder. There are two ways to measure d. It can be determined by measuring the changes of generated pressure due to the change in P_j. In practice, it has been more convenient to make the measurements by holding the generated pressure constant by adjusting the load on the piston as required by changes in P_j. The pressure can be monitored using any pressure measurement device of sufficient sensitivity and short-term stability. Often, a second piston gauge is used for this purpose. The slope of the curve obtained by plotting the applied force as a function of P_j is the value of d.

Both P_z and P_j are obtained from measurements of the fall rate $\delta s/\delta t$ which are related by[4]

$$(\delta s/\delta t)^{1/3}=K(P_z-P_j), \tag{4}$$

where K is a constant. The technique is to measure the fall rate at several values of P_j for each of several loads, plotting P_j as a function of the cube root of the fall rate, and extrapolating the linear portion of the curves to zero fall rate. The intercept of the P_j axis at zero fall rate for a given load is the value of P_z for the load. The extrapolation is done using a least-squares fitting routine. Then P_z can be expressed as a polynominal function of the load W as

$$P_z=P_{z_0}+S_zW+q_zW^2. \tag{5}$$

The appropriate values of P_j for gauge operation are also selected from the plot so as to give fall rates that are consistent with small clearance and reasonably low friction.

Each of the terms in Eq. (1) contributes to the overall uncertainty of the pressure calculation. The fractional uncertainty in P can be expressed as the root-mean-square of the total differential[4]

$$\frac{dP}{P}=\left[\sum_1^i\left(\frac{1}{P}\frac{\delta P}{\delta X_i}dX_i\right)^2\right]^{1/2}, \tag{6}$$

where X_i are the individual parameters and coefficients of Eq. (1). As an example, the values of the differentials, their uncertainties, and the fractional uncertainty in P for the gauge used in this study are listed in Table I.[9] The largest contributions to the uncertainty are due to A_0, P_z, and d. The determination of A_0 is a dimensional metrology problem. The values of P_z and d depend upon the conditions under which the gauge is used. In this paper, the effect of viscosity, temperature, and rate of rotation on the values of P_z are examined.

II. EXPERIMENT

During the present studies, a commercially available, 700-MPa controlled-clearance piston gauge was used. The piston is made from carbide and has a nominal diameter of 2.86606 ± 0.00007 mm. The cylinder is made of steel. The instrument was placed in a vibration-free room where the temperature could be adjusted from 290 to 303 K. A valve was provided at the bottom of the cylinder to isolate the gauge from the rest of the pressure system during fall rate measurements. The temperature of the gauge was measured within an inaccuracy of 0.1 K with a mercury thermometer in close thermal contact with the outer surface of the cylinder.

A bourdon tube gauge was used to measure P_j within an inaccuracy of ± 0.7 MPa. Fall rates were measured using a linear displacement dial gauge and a stop watch. The speed of rotation was determined using a photoelectric switch and a counter. The rotation rate could be adjusted between 177 and 425 rpm by using drive pulleys of various diameters.

The pressure transmitting fluids used in these experiments were Univis J-13 and mixtures of J-13 and heptane. In all, four fluids were used: 100% J-13, 50 vol. % heptane and 50% J-13, 65 vol. % heptane and 35% J-13, and 90 vol. % heptane and 10% J-13 which are denoted as H0,100; H50,50; H65,35; and H90,10, respectively. When changing fluids the entire pressure system including the gauge was thoroughly flushed with the newly prepared fluid before filling. Precautions were taken to remove any traces of the previously used fluid and to see that the air was completely expelled as the new fluid was installed. The densities of the fluids were measured before and after the fall rate measurements with no measurable change within the $\pm 0.1\%$ resolution of the densimeter. The constant density for a given fluid is an indication that the concentration of the heptane in the mixtures remained unchanged during the fall rate measurements.

Fall rate measurements were made in the region of the gauge reference level. About 30 min between two successive observations were adequate to allow the system to return to equilibrium. Starting with approximately 10% of the maximum possible load on the piston, P_j was increased until the time for the piston to fall through a fixed distance was long enough to be conveniently measured. Fall times then were measured with increasing P_j, using the same load on the piston, until the curve of P_j plotted as a function of the cube root of the fall rate deviated from linearity. The load was then increased and the fall rate measurements repeated for the new pressure. Measurements were made with both increasing and decreasing loads on the piston using all four fluids, varied rates of rotation, and different temperatures over the range of 290 to 303 K. The viscosity of all four fluids was also measured as a function of pressure up to 100 MPa at 297 K using a high-pressure rolling ball viscometer.

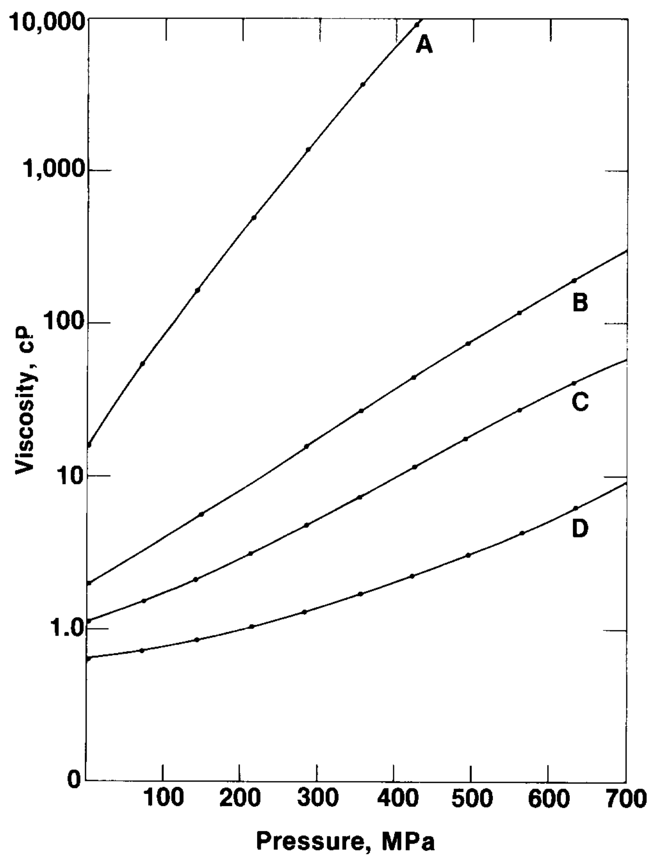

FIG. 1. Viscosity as a function of pressure for the four fluids H0,100 (A), H50,50 (B), H65,35 (C), and H90,10 (D).

III. RESULTS

The viscosity of the four fluids is plotted as a function of pressure in Fig. 1. To first order, the viscosity is a logarithmic function of pressure. H90,10 is the least viscous fluid and also has the smallest pressure dependence. Viscosity and the pressure dependence both increase monotonically as the percentage of J-13 is increased.

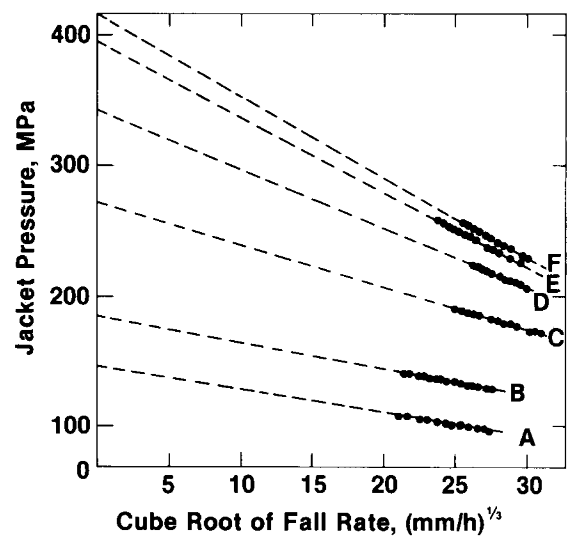

FIG. 2. Jacket pressure as a function of the cube root of the fall rate for H0,100 at 177 rpm and 297 K. Curves A–F are for 82, 150, 290, 427, 563, and 703 MPa, respectively. The intercepts of the P_j axis are the values of P_z for those pressures.

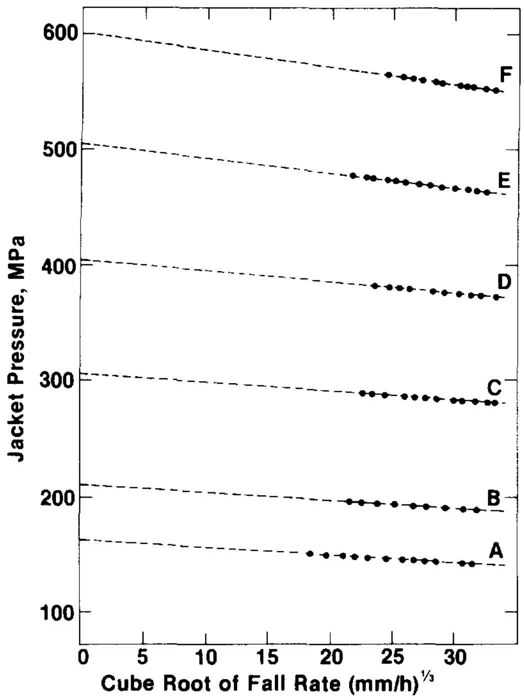

FIG. 3. Jacket pressure as a function of the cube root of the fall rate for H90,10 at 177 rpm and 297 K. Curves A–F are for 82, 150, 290, 427, 563, and 703 MPa, respectively. The intercepts of the P_j axis are the values of P_z for those pressures.

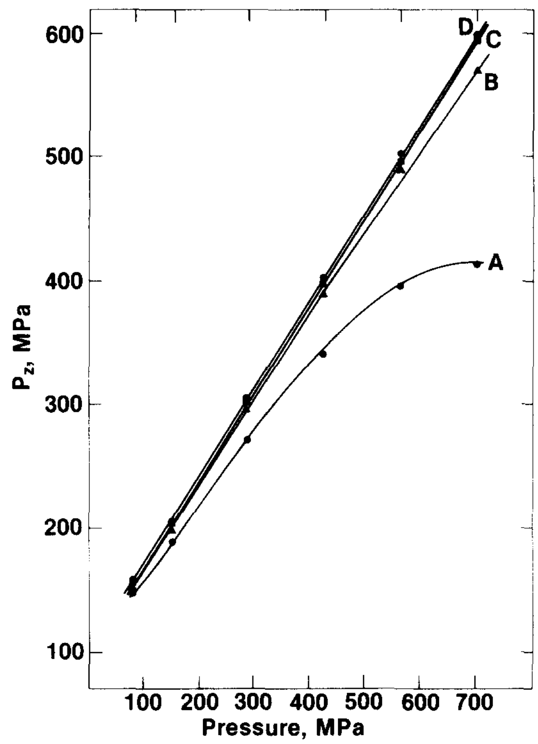

FIG. 5. P_z as a function of pressure for the four fluids H0,100 (A), H50,50 (B), H65,35 (C), and H90,10 (D) at 177 rpm and 297 K.

The fall rate curves for H0,100 and H90,10 obtained at 297 K and 177 rpm are shown in Figs. 2 and 3, respectively. These two fluids represent the extremes of the viscosities in this study. The spacing between two fall rate curves corresponding to two successive loads is greater for H90,10, while the change in the slope of the curves with increasing load is greater for H0,100. The fall rate curve for H50,50 and H65,35 are not shown but are consistent with the results of the other two fluids. The slopes of the fall rate curves for all four fluids are plotted in Fig. 4 as a function of pressure.

The values of P_z for all four fluids are plotted as a function pressure in Fig. 5. For H90,10 and H65,35, the function is linear. For H50,50 the relationship becomes nonlinear in the highest 50% of the pressure range. The nonlinearity is greatest for H0,100.

A series of fall rate measurements at approximately 20%, 40%, and 60% full-scale pressure were done at temperatures of 291, 297, and 302 K using all four fluids with the piston rotating at 177 rpm. In Fig. 6, P_z is plotted as a function of temperature for the three pressures for H90,10 and H0,100. Figure 7 is a plot of P_z as a function of pressure for the three temperatures for the same two fluids. The results for H65,35 and H50,50 are consistent with those of the other

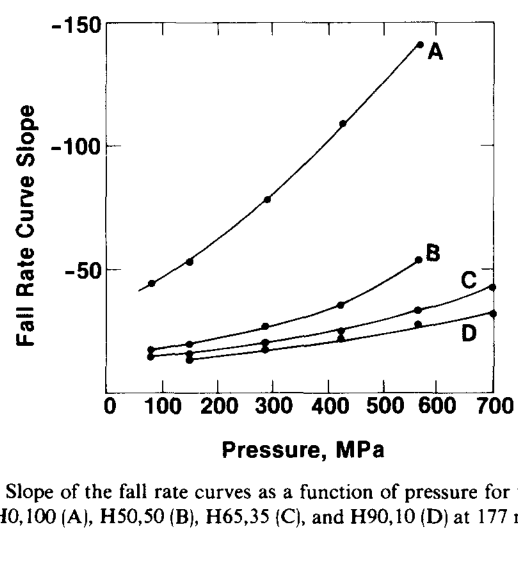

FIG. 4. Slope of the fall rate curves as a function of pressure for the four fluids H0,100 (A), H50,50 (B), H65,35 (C), and H90,10 (D) at 177 rpm and 297 K.

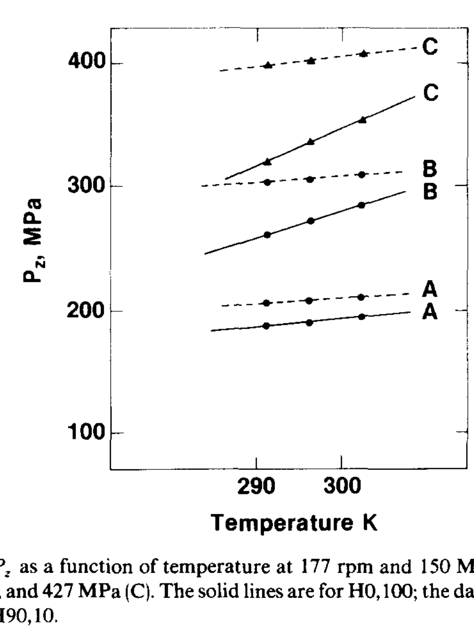

FIG. 6. P_z as a function of temperature at 177 rpm and 150 MPa (A), 290 MPa (B), and 427 MPa (C). The solid lines are for H0,100; the dashed curves are for H90,10.

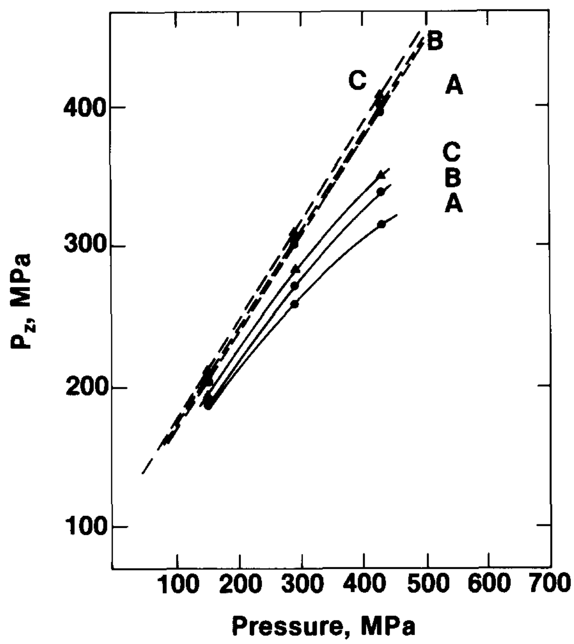

FIG. 7. P_z as a function of pressure at 177 rpm and 291 K (A), 297 K (B), and 302 K (C). The solid lines are for H0,100; the dashed curves are for H90,10.

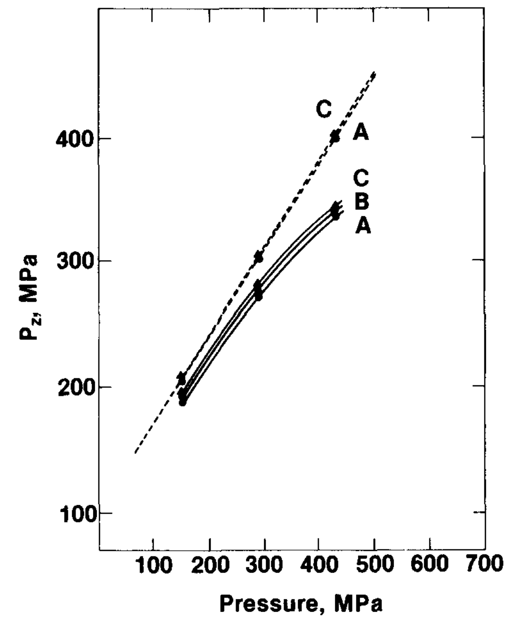

FIG. 9. P_z as a function of pressure at 297 K and 177 rpm (A), 325 rpm (B), and 425 rpm (C). The solid lines are for H0,100; the dashed curves represent H90,10.

two fluids and have not been plotted in Figs. 6 and 7 for the sake of clarity.

Another series of fall rate measurements was done at 297 K at approximately 20%, 40%, and 60% full-scale pressure using all four fluids with the piston rotating at 177, 325, and 425 rpm. The resulting values of P_z are plotted as a function of speed of rotation for each pressure for H0,100 and H90,10 in Fig. 8. In Fig. 9, P_z is plotted as a function of pressure for each rotation rate for the same two fluids. Again, the values for H65,35 and H50,50 have been omitted for clarity.

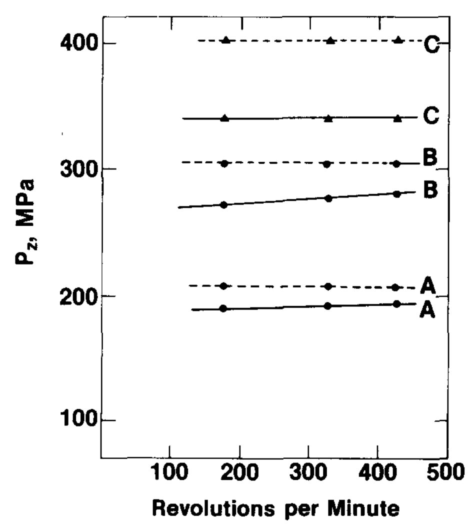

FIG. 8. P_z as a function of rotation rate at 297 K and 150 MPa (A), 290 MPa (B), and 427 MPa (C). The solid lines are for H0,100; the dashed curves represent H90,10.

IV. DISCUSSION

An expression for the slope of the fall rate curve m is given in Ref. 8 as

$$m = C(\mu L/P)^{1/3}, \tag{7}$$

where C is a constant, μ is the viscosity at pressure, and L is the length of the path between the piston and cylinder over which the flow gradient takes place. Furthermore, it was suggested that if L can be considered to be a constant, then the viscosity can be obtained from fall rate data. Assuming L to be a constant, then we can write

$$C' = CL^{1/3} = m(P/\mu)^{1/3}, \tag{8}$$

where C' is constant. In order to test the validity of this assumption, C' has been calculated for all four fluids using the viscosity data of Fig. 1 and the fall rate curve slope data of Fig. 4. The resulting values of C' are plotted as a function of pressure in Fig. 10. It is abundantly clear that C' is not a constant independent of P or μ. However, since the slope of the C' curve for H0,100 is positive and for all the other fluids it is negative, there should exist a fluid for which C' is a constant as a special case.

Since C' is not a constant, then either L is not constant or some assumption in the derivation of Eq. (7) is not valid. If L is, indeed, not constant, then there is an important consequence to consider. Since L is the length of the path between the piston and cylinder over which the flow gradient takes place, if L is not constant, then there is a high probability that d, the coefficient describing the distortion of the cylinder, is also not constant. Values for d can be calculated using Eq. (3) or can be measured using the methods outlined earlier. The calculation yields a constant value for d. The measurements, in general, result in an expression of d as a function of pressure. Until reexamination of Eq. (7) is completed,

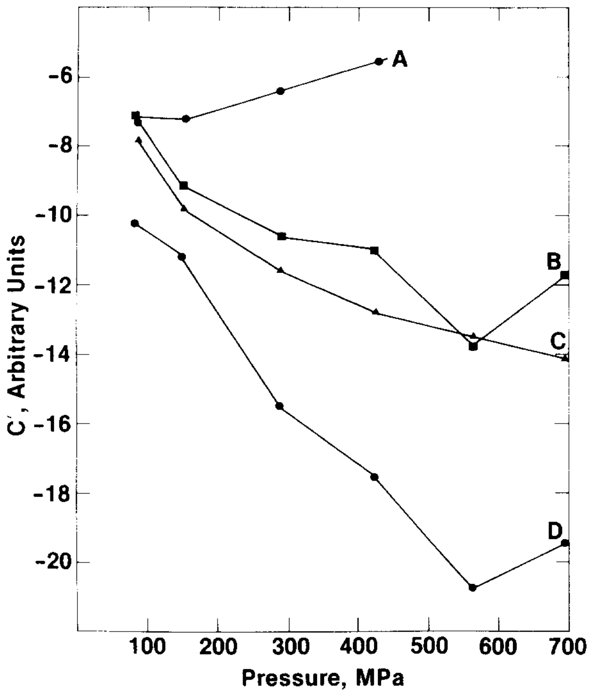

FIG. 10. C' as a function of pressure for H0,100 (A), H50,50 (B), H65,35 (C), and H90,10 (D).

it is prudent to measure d rather than to calculate it.

A reexamination of Eq. (7) may be complicated by a lack of understanding of viscosity in such thin fluid films as are found in piston gauges. Typically, the clearance between the piston and cylinder is on the order of 2 μm. That the behavior of the viscosity in this narrow space is unusual is indicated by the data of Figs. 1 and 5. In Fig. 5, P_z is plotted as a function of pressure for all four fluids. The curves for H90,10, H65,35, and the lower half for H50,50 are linear and almost identical. In the region where these curves are so similar, the viscosities for the three fluids, according to Fig. 1, vary from 0.7 to 60 cP, a factor of 85. The viscosity of H50,50 spans the range from 2 to 132 cP. In the region of 60 cP, the curve in Fig. 5 starts to deviate from linearity. In the case of H0,100, only the lowest point is in approximate agreement with the other three fluids. The viscosity corresponding to this point is about 63 cP. This evidence tends to suggest that the operation of this piston gauge is nearly independent of viscosity over a wide range up to some critical value (on the order of 60 cP) at which point the viscosity begins to have a very large influence.

Additional evidence that the operation of other piston gauges may be independent of viscosity over some range is found in other recent results,[10] wherein a simple piston gauge was modified to provide for the measurement of pressure in the annular space between the piston and cylinder along the cylinder working length. The pressure profile so obtained using two oils differing in viscosity by a factor of 2.4 were essentially identical. Furthermore, a theory for calculating the pressure profile is in excellent agreement with the measurements. The theory regards viscosity to be a constant while, in fact, it changed by a factor of 2 for both oils over the pressure range of the study. Clearly, the understanding of the role of viscosity of the working fluid in piston gauges is not yet complete.

That P_z is dependent upon the temperature is indicated in Figs. 6 and 7. Two ways in which the temperature could effect the fall rates, and hence P_z, are through the thermal expansion of the piston and cylinder and through the temperature dependence of the viscosity.

Over the temperature range considered here, the viscosity at atmospheric pressure of heptane changes by 0.005 cP/°C, while that of J13 changes by 0.62 cP/°C. Under the assumption that the temperature change will shift the viscosity curves of Fig. 1 without changing the slope appreciably, the maximum viscosity for H90,10 for the temperature and pressure involved with Figs. 6 and 7 is less than 3 cP while the minimum for H0,100 is greater than 170 cP.

Since P_z appears to be relatively insensitive to viscosity over the range of 0.7 to 60 cP, the temperature effects on P_z when using H90,10 are probably due largely to the thermal expansion of the piston and the cylinder. The effect is a function of pressure. The highest pressure in this study, and hence the worst case, is 427 MPa. At 427 MPa and upon cooling from 296 to 291 K, the change in effective area due to the P_z thermal effect is 26 ppm. When heated from 296 to 302 K at the same pressure, the change in the effective area due to the P_z thermal effect is 37 ppm.

The case of H0,100 is more complicated because the P_z thermal effect involves both thermal expansion and viscosity. That the effect is much more nonlinear than H90,10 is evident in Fig. 6 by the change of the slopes of curves A, B, and C for H0,100, and in Fig. 7 by the disproportionate separation between curves A and B for a change of 5 K compared to that of B and C for a change of 6 K. For the worst case, at 427 MPa, the change in effective area due to the P_z thermal effect for H0,100 is 168 ppm upon cooling to 291 K and 70 ppm upon heating to 302 K.

There are four practical methods to deal with the temperature effect of P_z: (1) Always use the piston gauge at the same temperature at which it was characterized. (2) Characterize the piston gauge over a range of temperatures and, thereby experimentally determine temperature-dependent terms to be included in expressions for P_z, such as Eq. (5). (3) If a choice of pressure fluids is possible, the P_z thermal effect will be reduced by using the fluid having the lowest viscosity at pressure. (4) Increase the uncertainty pertaining to P_z to accommodate expected temperature changes.

The change in P_z with rate of piston rotation is seen in Figs. 8 and 9. The measurements were made at three rotating rates: 177, 325, and 425 rpm. The curves in these two plots are smooth and well behaved, free of the rather unusual effects reported by Newhall, Ogawa, and Zilberstein[8] for DTE-24 oil at rotation rates under 50 rpm. Since P_z has a measurable dependence on the rate of rotation, it is important to use the piston gauge at the same rotation rate at which it was characterized.

The value of P_z, and hence the effective area of a controlled-clearance piston gauge, has a clearly demonstrable dependence on the fluid viscosity, temperature, and the rate of piston rotation. The dependence of P_z on the viscosity is slight up to a critical value in the neighborhood of 60 cP.

Beyond 60 cP, the effects of viscosity are increasingly more pronounced. As both the temperature dependence and the rate of rotation dependence of P_z are greatest for the fluid of greatest viscosity, normally one would wish to choose a fluid having the lowest viscosity at pressure. For pressure metrology laboratories not equipped with a high-pressure viscometer, a practical guide for selecting a candidate fluid is to choose the fluid having the most nearly linear relationship between P_z and pressure, as is suggested by Fig. 5.

The dependence of P_z upon viscosity, temperature, and rate of rotation stresses the need to use piston gauges under the identical conditions prevailing during characterization.

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